Math 8061, Homework 10

(Can be handed in by Thursday 12/2/21 for extra credit)

0. Read the proof of Theorem 11.49, the Poincaré Lemma for 1–forms. A version of the same argument works for p–forms.

1. Suppose that ω is a closed form on M, and η is exact. Prove that $\omega \wedge \eta$ is exact.

2. Let M be a smooth n–manifold, and $f_1, \ldots, f_n$ smooth functions from M to $\mathbb{R}$. Prove that $(f_1, \ldots, f_n) : M \to \mathbb{R}^n$ defines a set of coordinates near $p \in M$ if and only if the n–form $df_1 \wedge \cdots \wedge df_n$ is non-vanishing at p.

3. Suppose M and N are connected, oriented smooth n–manifolds, and $f : M \to N$ is an immersion. Prove that f is orientation–preserving everywhere or orientation–reversing everywhere.

4. Problems on orientations:

a) If M is orientable, prove that an open set $U \subset M$ is orientable.

b) If M and N are orientable, prove that $M \times N$ is orientable.

c) If M is not orientable, prove that $M \times N$ is not orientable for any N. *Hint:* Use part (a) and induction on dimension.