

## Math 8061 Homework 3

Due Thursday, 9/30/10

1. Let  $M$  be a smooth compact  $n$ -manifold. Prove that there is no submersion  $f : M \rightarrow \mathbb{R}^k$ , for any  $k > 0$ .

2. If  $M$  is compact, prove that every 1-1 immersion  $f : M \rightarrow N$  is an embedding.

3. Let  $\mathbb{RP}^2 = S^2 / \sim$ , where antipodal points on the sphere are identified.

a) Prove that the map  $g : \mathbb{RP}^2 \rightarrow \mathbb{R}^4$ , defined by

$$g([x, y, z]) = (yz, xz, xy, x^2 - y^2),$$

is a smooth embedding.

b) Prove that the map  $g : \mathbb{RP}^2 \rightarrow \mathbb{R}^3$ , defined by

$$g([x, y, z]) = (yz, xz, xy)$$

fails to be an immersion at 6 points (the 6 image points are at distance  $1/2$  from the origin, on each axis). In fact, there is an immersion  $h : \mathbb{RP}^2 \rightarrow \mathbb{R}^3$ , called Boy's surface. See

[http://en.wikipedia.org/wiki/Boy's\\_surface](http://en.wikipedia.org/wiki/Boy's_surface)

4. Do problem 27, on page 60 of Spivak.

5. Do problem 32, on page 61 of Spivak.