Math 8051 Homework 6

Due Wednesday, 10/25/17

1. Page 83, Exercise 3. *Hint:* use partial fractions.

2. Let $f : \mathbb{C} \to \mathbb{C}$ be a continuous function that is analytic on $\mathbb{C} \setminus \mathbb{R}$. Use Morera's theorem to prove that f is analytic on $\mathbb{C}$, i.e. entire.

3. Page 96, Exercise 8.

4. Page 96, Exercise 11. *Hint:* use the Cauchy Integral Formula. Alternately, modify each of the curves by a homotopy in $\mathbb{C} \setminus \{0\}$ until it becomes a circle, possibly traversed more than once.

3. Let G be a region. Let $\gamma_0, \gamma_1 : [0,1] \to G$ be (piecewise smooth) closed curves. A *free homotopy* from γ_0 to γ_1 is a (piecewise smooth) function $\Gamma : [0,1] \times [0,1] \to G$ such that

- $\Gamma(s,0) = \gamma_0(s), \quad \Gamma(s,1) = \gamma_1(s) \quad \forall s.$
- $\Gamma(0,t) = \Gamma(1,t) \quad \forall t.$

The main difference from a fixed-endpoint homotopy is that the endpoints of $\gamma_t(s) = \Gamma(s,t)$ are allowed to move with t; however, $\gamma_t(s)$ must be a closed curve for all t. Compare Definition 6.1 in the book.

Prove that if $f : G \to \mathbb{C}$ is analytic and γ_0, γ_1 are freely homotopic loops, then $\int_{\gamma_0} f = \int_{\gamma_1} f$. *Hint:* construct a closed loop γ_2 that is fixed-endpoint homotopic to γ_0 and also satisfies $\int_{\gamma_1} f = \int_{\gamma_2} f$. An alternate approach is to modify the endgame in the proof of the Independence of Path Theorem from class.