Extra Problems for Homework 3

Math 461, Fall 2006

The following problems are part of the assigned homework for graduate students, and are extra credit for undergrads.

1. Let $\mathcal{B}$ be a basis for a topology τ on X.
(a) Prove that τ is the intersection of all topologies that contain $\mathcal{B}$.
(b) Prove that the same conclusion holds when $\mathcal{B}$ is a subbasis.

2. For the integers $\mathbb{Z}$, let $\mathcal{B}$ be the set of all arithmetic sequences that extend infinitely far in both directions. For example, one element of $\mathcal{B}$ is

$$B = \{\ldots, -3, 1, 5, 9, \ldots\}.$$

(a) Prove that $\mathcal{B}$ is a basis for a topology τ on $\mathbb{Z}$.
(b) Describe the simplest closed sets (apart from $\mathbb{Z}$ and $\emptyset$) that you can think of in this topology.
(c) Use the topology τ to prove that there are infinitely many prime numbers.