Study Guide for Final Exam

Math 4096, Spring 2020

1. You should know the definitions of the following terms.

- knot, link
- Reidemeister moves
- the connected sum of two knots
- prime/composite knot
- split link
- the crossing number of a knot or link
- homeomorphism
- isotopy (of links or surfaces)
- Euler characteristic
- orientability of a surface
- the genus of a knot or link
- torus knot
- satellite knot
- span of a polynomial

2. Know how to compute or construct the following:

- tricolorabiliity
- linking number
- going from a diagram to its Dowker code and back
- Euler characteristic of a surface
- Seifert surface of a knot or link
- A satellite knot, given a pattern and a companion
- a diagram in braid position, given an arbitrary diagram of K
- a closed braid from a word in the σ_i's
- bracket polynomial
- writhe
- Jones polynomial

3. Theorems to know:

- Reidemeister's theorem about Reidemeister moves

- Classification of surfaces by orientability, genus, and number of boundary components

- The Euler characteristic of an (orientable) surface of genus g is $\chi(S) = 2 - 2g$.

- Genus is additive under connected sum

- Three types of knots: torus, satellite, hyperbolic

- Alexander's theorem about putting links in braid position

- The span of the Jones polynomial gives the crossing number of an alternating link

4. Do the following quantities depend on a diagram? Or are they link invariants?

- number of crossings

- number of link components

- Dowker code

- tricolorability

- genus of a Seifert surface

- writhe

- bracket polynomial

- span of the bracket polynomial

5. There will be several statements that you will have to classify as True, False, or Unknown. Here, *true* means "true in all circumstances," and *unknown* means that humanity does not know if it is true or false. For statements that are false, you should give a counterexample. For statements that are true, you should give a short explanation (but less than a full proof).
Some examples:

- A diagram without nugatory crossings has the smallest possible number of crossings for that link.

- The crossing number is additive under connected sum.

- Every knot has an alternating diagram.

- The unknot is prime.

- Orientability of a surface is preserved by homeomorphisms.

- The large majority of knots are hyperbolic.