Knot theory project guidelines

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Important Checkpoint Dates.

- **Thu Feb 6:** Potential group members & general area of interest, communicated to me. From this information I will finalize the groups.
- **Thu Feb 27:** Specific topic and initial reference list communicated to me.
- **Tue Mar 24:** First draft of paper due.
- **Thu-Fri, Mar 26-27:** Individual consultations about revisions
- **Tue Apr 14 – Thu Apr 23:** In–class presentations
- **Thu Apr 23:** Final version of Paper due.
- **Thu Apr 23:** Individual Reflection due (each member of group completes their own reflection).

Purpose of project. The purpose of the project is for you to learn about an area or topic in mathematics related to knot theory that you find interesting. It can be an extension or deeper study of a topic that we discuss in class or something related to knot theory not discussed in class. Once you and your group has decided what kind of topic you would like to study, you will find resources, narrow down your topic, write a paper, and give a presentation on your work to the class.

In short, the project will allow for you to practice working collaboratively on mathematics in an area of interest to you. You will also gain experience writing a technical paper and giving a technical presentation.

Your Paper. Your paper should be about 10 pages long, with proper sectioning and numbered equations (if equations are needed). The paper should include an abstract and bibliography.

Depending on your topic and diagrams included, the final paper might be 8–12 pages long, but should not be shorter that 8 pages or longer than 12 pages. The paper should be typeset using Word or LaTeX, and converted to a pdf for final submission. If using LaTeX, the website `https://www.overleaf.com` might be a useful tool for collaboration.

Your Presentation. Over the course of the last two weeks of the semester, each group will give a 20–minute presentation on their work in class. Each student in your group should present for roughly one–third of the time. All presentations must be presented using PowerPoint, the Beamer presentation package in LaTeX, or some other digital presentation medium that I approve. The specific dates will be determined once the topics been chosen.

The goal of the presentation is to introduce the other students in the class to the topic that you studied. You will not have time to talk about every detail covered in your paper, so it will take some work to decide how to most clearly deliver the content that you wish the audience to absorb.

The audience is encouraged to participate in the presentation (constructively, of course). You should be able to answer reasonable questions about the topic from your fellow students or from me.

Your Individual Reflection. Each person will submit their own Individual Reflection, a one page reflection that contains a summary of what you have learned through completing the project. Things you should address in your reflection:

- How did the structure of the project help you learn about your topic?
- What were your specific contributions to the project?
- Do you feel your contributions were significant?
- What strengths did your group have?
- What challenges did you encounter completing the work as a group, or areas you could improve on?
- Did your group work well together?

A Note About Group Work. You must work in groups of two or three people on the project. How you divide up the work is up to you, but each member should contribute an equal amount to the overall project, as each group member will receive the same grade, *except for the grade on the individual reflection.*

One way to divide the work is to have a group member who is the coordinator for a certain part of the project. For example, a group of three may choose to have one person in charge of getting references from the library, one person takes the lead on the first draft of the paper, and the third person takes the lead on the first draft of the presentation (determines what tools to use, structure of presentation, or if a video is chosen as presentation media does the actual presentation). The other members of the group will contribute and offer input on these tasks, but the coordinator is ultimately in charge of getting it done. **All group members will contribute equal effort to the scholarly process of creating the information that will go in the paper and presentation.**

Keep in touch with the members of your group throughout the semester, and ensure that progress is being made on your project (clarifying the question, finding references, getting a rough draft written, etc.)! **Turning a task over to someone and not getting reports from them on their progress until just prior to one of the checkpoint dates is a recipe for disaster.** Even though you may not be coordinator for a task, you should be seeing progress on it. Also, if you are the coordinator and are having difficulty with something, the first thing to do is talk to the rest of your group about it to see if they can help you. Regular meetings throughout the semester are a requirement to ensure your project stays on track. Your group (in whole or in part) can also come see me when you have questions about your topic.

Suggested projects. Here are some suggested topics. You are encouraged to mutate these into problems that interest you more! If you want to know more about some of these topics, come talk to me or search the internet.

- **Topic of your choice:** Find a topic related to knot theory of our choosing. Please run your idea by me early so that I can help you focus your idea and ensure that it is not going to be too difficult. Try doing a web search for "knot theory and ⟨keyword⟩" to try to find connections between knot theory an other topics you're interested in.
- **Unsolved problems:** Study one of the unsolved problems listed in the textbook. What has been done? What is the most up-to-date answer? Can you work out some examples?
- **Knots and links in graphs:** Embeddings of non–planar graphs often contain knots and links. Explore this. See chapter 8 of the textbook to get started. Also see `http://people.reed.edu/~ormsbyk/138/ConwayGordon.pdf`.
- **Gauss's linking number integral:** Fifty years or so before Tait and friends started studying knot theory, Gauss described an integral that computes the linking number. Explore this and other formulations of linking number. See `http://www.maths.ed.ac.uk/~aar/papers/ricca.pdf` to get started.
- **Algorithmic problems in knot theory:** Explore the algorithmic approaches to deciding knot theoretic problems. Studying the unknot recognition problem is one route, but there are many others. Here is a good starting point: `https://arxiv.org/abs/1604.03778`. This paper lists many problems and references; you may wish to explore these references.
- **Crossing numbers of composite knots:** Study the crossing number conjecture. What is its history? What progress has been made? What tools or techniques are used? Here is a good starting point: `http://front.math.ucdavis.edu/0805.4706`.
- **The fundamental group and knots:** The fundamental group is an extremely powerful invariant of knots. Learn about the fundamental group. What is a group presentation? How can we compute the fundamental group combinatorially? Can you use the fundamental group to distinguish some of the knots we've talked about in class? *Introduction to knot theory* by Cromwell and Fox is one (of many) good resources for this.
- **Braids, braid groups, Artin groups**: Learn about braids. Learn about their associated groups. See section 5.4 of the textbook to get started.
- n–**Colorability:** This is a generalization of tricolorability using many colors. It has connections to fundamental groups and Jones polynomials. This paper provides a reference: `http://matwbn.icm.edu.pl/ksiazki/bcp/bcp42/bcp42120.pdf`.
- **Lorentz knots:** The Lorentz equations are a system of differential equations. Some of the solutions to these differential equations produce beautiful knots in $\mathbb{R}^3$. Study these knots and their connections. to braids, differential equations, and other topics. See `http://arxiv.org/pdf/1201.0214.pdf` to get started.
- **Ropelength:** What is the minimal length of rope of diameter 1 cm needed to tie a given knot? This is a deceptively hard question. Much research has been done on this problem.

See the following and its references to get started: http://torus.math.uiuc.edu/jms/Papers/thick/ropelen.pdf.

- **Knot tabulation:** Learn the details of a knot tabulation method. Hoste, Thistlethwaite, and Weeks have a more computational method. Conway has a more combinatorial method. Can you write a program duplicating their results up to some number of crossings? Can you do better? What is the history and current status of knot tabulation? See http://pzacad.pitzer.edu/~jhoste/HosteWebPages/downloads/HTW.pdf and http://www.maths.ed.ac.uk/~aar/papers/conway.pdf to get started.
- **Random knots and links:** Instead of asking about all knots up to a given number of crossings, it is often fruitful to investigate "generic" or "random" knots. What does a "typical" knot look like? How are the invariants of random knots distributed? There is a large literature on this subject. This paper provides a point of entry: https://arxiv.org/pdf/1608.02638.pdf
- **Knot theory applied to DNA:** Knotting occurs in DNA and there is interaction between knot theorists and biologists. Learn about how knot theory and DNA interact. See chapter 7 of the textbook. Also see the book *Applications of Knot Theory*, by Buck and Flapan.
- **Knot theory and physics:** There are various ways in which knot theory interacts with physics. The discussion in the textbook of knot theory and statistical physics is one jumping off point. Louis Kauffman has a nice book called *Knots and Physics* that contains many possible project ideas. The library has this book.
- **Knot theory and cryptography:** Various computer scientists and mathematicians have proposed a cryptographic system based on the difficulty of factoring complicated knots into prime factors. Learn about this cryptosystem, its strengths and weaknesses, as well as other related approaches. This paper is a good place to start: https://arxiv.org/pdf/1206.5709v1.pdf.

Grading scheme.

- 40% for depth and mathematical accuracy. How challenging is the topic? How well did your group choose examples to illustrate the concept? How correct are the mathematical statements in the paper?
- 30% for the writing quality of the paper. Does it have appropriate mathematical formatting, with sections, displayed formulas, etc? Is the writing well ordered and organized? Is it clear and easy to follow?
- 20% for the presentation. Is the presentation clear and well explained? Does it use visual tools effectively? Does it have clear examples? Is it well paced? Did you address questions from the audience?
- 10% for the individual reflection, promptness in the various deadlines, and participation during the presentations by other groups.