

Study Guide for Final Exam

Math 4096, Fall 2016

1. You should know the definitions of the following terms.

- knot, link
- Reidemeister moves
- the connected sum of two knots
- prime/composite knot
- split link
- the crossing number of a knot or link
- homeomorphism
- isotopy (of links or surfaces)
- Euler characteristic
- orientability of a surface
- the genus of a knot or link
- torus knot
- satellite knot
- span of a polynomial

2. Know how to compute or construct the following:

- tricolorability
- linking number
- going from a diagram to its Dowker code and back
- Euler characteristic of a surface
- Seifert surface of a knot or link
- A satellite knot, given a pattern and a companion
- a closed braid from a word in the σ_i 's
- bracket polynomial
- writhe
- Jones polynomial
- Alexander polynomial

3. Theorems to know:

- Reidemeister's theorem about Reidemeister moves
- Classification of surfaces by orientability, genus, and number of boundary components
- The Euler characteristic of an (orientable) surface of genus g is $\chi(S) = 2 - 2g$.
- Genus is additive under connected sum
- Three types of knots: torus, satellite, hyperbolic
- Alexander's theorem about putting links in braid position
- The span of the Jones polynomial gives the crossing number of an alternating link

4. Do the following quantities depend on a diagram? Or are they link invariants?

- number of crossings
- number of link components
- Dowker code
- tricolorability
- genus of a Seifert surface
- writhe
- bracket polynomial
- span of the bracket polynomial

5. There will be several statements that you will have to classify as True, False, or Unknown. Here, *true* means "true in all circumstances," and *unknown* means that humanity does not know if it is true or false. For statements that are false, you should give a counterexample. For statements that are true, you should give a short explanation (but less than a full proof).

Some examples:

- A diagram without nugatory crossings has the smallest possible number of crossings for that link.
- The crossing number is additive under connected sum.
- Every knot has an alternating diagram.
- The unknot is prime.
- Orientability of a surface is preserved by homeomorphisms.
- The large majority of knots are hyperbolic.