

Study Guide for Final Exam

Math 4061, Spring 2010

1. You should know the definitions of the following terms. Several of them will appear on the test.

- simple closed curve
- tangent, normal, binormal vectors
- curvature of a curve
- tangent plane to surface
- orientable surface
- principal curvatures, principal vectors
- Gaussian curvature
- elliptic, hyperbolic, parabolic, planar points of a surface
- umbilic point
- geodesic
- Euler characteristic

2. Know how to compute the following:

- length of a curve
- curvature and torsion for a *unit-speed* curve
- normal vector to a surface
- the area of a region on a surface, using the first fundamental form matrix \mathcal{F}_I
- the principal curvatures and principal vectors, using the matrices \mathcal{F}_I and \mathcal{F}_{II}

You do **not** need to memorize the formulas for \mathcal{F}_I and \mathcal{F}_{II} : they will be provided if needed.

3. Theorems to know:

- A curve is determined up to rigid motions by its curvature and torsion.
- The integral of signed curvature over a closed curve is 2π times the winding number.
- Euler's theorem about normal curvature
- Gauss–Bonnet theorem for disk-like regions in a surface
- Gauss–Bonnet theorem for compact surfaces

4. Are the following properties of surfaces preserved by (a) all homeomorphisms, (b) all diffeomorphisms, (c) all isometries, (d) all rigid motions, or (e) not necessarily preserved by any of the above?

- (c) area
- (d) principal curvatures
- (c) Gaussian curvature
- (a) Euler characteristic
- (b) orientability
- (c) the property that a particular curve is a geodesic
- (d) the normal curvature along a curve
- (e) the normal vector to a curve
- (c) length of curves
- (b) the index $\mu(P)$ of a stationary point of a vector field

5. Are the following statements true or false?

- (F) In polar coordinates on \mathbb{R}^2 , the curve $r = 1 + 2 \cos \theta$ is a simple closed curve.
- (T) Let p be a point of a surface where the Gaussian curvature is -1 . Then p cannot be an umbilic point.
- (F) The pseudosphere is compact.
- (F) If a tangent plane $T_p S$ intersects S at only the point p , then p is an elliptic point.
- (F) There is a compact surface $S \subset \mathbb{R}^3$ whose Euler characteristic is $\chi(S) = 1$.
- (F) On the pseudosphere, there is a triangle with geodesic sides whose area is 4.