# Review Questions for Midterm Exam

Math 4061, Spring 2010

**1.** You should know the definitions of the following terms. 4 of them will appear on the test.

- smooth curve
- regular curve
- simple closed curve
- tangent, normal, binormal vectors
- curvature (signed and unsigned)
- torsion
- surface patch/chart
- smooth surface
- tangent plane to surface
- orientable surface

**2.** For what class of curves in $\mathbb{R}^3$ do the statements below hold true? Is it (a) all curves in $\mathbb{R}^3$, (b) smooth curves in $\mathbb{R}^3$, (c) regular curves in $\mathbb{R}^3$, (d) unit-speed curves in $\mathbb{R}^3$, or (e) none of the above?

- The tangent vector is defined.
- The first and second derivatives are perpendicular: $\gamma'(t) \cdot \gamma''(t) = 0$.
- Curvature is defined.
- Torsion is defined.
- $\gamma$ has a unit-speed reparametrization.
- $\gamma$ is determined up to rigid motions by its curvature and torsion.

**3.** Let $\gamma(t) = (t, \cosh t)$ for $t \in \mathbb{R}$. (Recall $\cosh t = (e^t + e^{-t})/2$ is the hyperbolic cosine.)

- Calculate the arclength of $\gamma$ between points $(0, 1)$ and $(t, \cosh t)$.
- Compute the curvature of $\gamma$.

**4.** Let $\gamma(t) = \left(\frac{1}{3}(1+t)^{3/2}, \frac{1}{3}(1-t)^{3/2}, \frac{t}{\sqrt{2}}\right)$. Check that $\gamma$ is a unit-speed curve, and compute its curvature, torsion, $T$, $N$, and $B$.

**5.** Are the following statements about surfaces true or false?

- If $\Phi$ is a transition function between two charts, the Jacobian $D\Phi$ is an invertible matrix.
- The tangent plane $T_pS$ depends on the choice of chart at $p$.
- The normal vector $\sigma_u \times \sigma_v$ depends on the choice of chart at $p$.
- The intersection between two smooth surfaces is a union of smooth curves.

**6** Let $S$ be the surface defined by $z = x\, e^{y/x}$, where $x > 0$. Prove that all tangent planes to $S$ pass through the origin.

**7.** Let $S$ be a surface with the following properties. Every pair of points $p, q \in S$ are connected by a smooth curve $\gamma$ lying in $S$. Also, for every smooth curve $\gamma$ on $S$, $\gamma(t) \cdot \gamma'(t) = 0$. Prove that $S$ is a sphere centered at the origin.