

Review Questions for Midterm Exam

Math 4061, Spring 2010

1. You should know the definitions of the following terms. 4 of them will appear on the test.
 - smooth curve
 - regular curve
 - simple closed curve
 - tangent, normal, binormal vectors
 - curvature (signed and unsigned)
 - torsion
 - surface patch/chart
 - smooth surface
 - tangent plane to surface
 - orientable surface

2. For what class of curves in \mathbb{R}^3 do the statements below hold true? Is it (a) all curves in \mathbb{R}^3 , (b) smooth curves in \mathbb{R}^3 , (c) regular curves in \mathbb{R}^3 , (d) unit-speed curves in \mathbb{R}^3 , or (e) none of the above?
 - (b) The tangent vector is defined.
 - (d) The first and second derivatives are perpendicular: $\gamma'(t) \cdot \gamma''(t) = 0$.
 - (c) Curvature is defined.
 - (e) Torsion is defined.
 - (c) γ has a unit-speed reparametrization.
 - (d) γ is determined up to rigid motions by its curvature and torsion.

3. Let $\gamma(t) = (t, \cosh t)$ for $t \in \mathbb{R}$. (Recall $\cosh t = (e^t + e^{-t})/2$ is the hyperbolic cosine.)
 - Calculate the arclength of γ between points $(0, 1)$ and $(t, \cosh t)$.
 - Compute the curvature of γ .

4. Let $\gamma(t) = \left(\frac{1}{3}(1+t)^{3/2}, \frac{1}{3}(1-t)^{3/2}, \frac{t}{\sqrt{2}}\right)$. Check that γ is a unit-speed curve, and compute its curvature, torsion, T , N , and B .

5. Are the following statements about surfaces true or false?

(T) If Φ is a transition function between two charts, the Jacobian $D\Phi$ is an invertible matrix.

(F) The tangent plane $T_p S$ depends on the choice of chart at p .

(T) The normal vector $\sigma_u \times \sigma_v$ depends on the choice of chart at p .

(T) The intersection between two smooth surfaces is a union of smooth curves.

6 Let S be the surface defined by $z = x e^{y/x}$, where $x > 0$. Prove that all tangent planes to S pass through the origin.

7. Let S be a surface with the following properties. Every pair of points $p, q \in S$ are connected by a smooth curve γ lying in S . Also, for every smooth curve γ on S , $\gamma(t) \cdot \gamma'(t) = 0$. Prove that S is a sphere centered at the origin.