Math 4061 Homework 8

Due Thursday, 4/29/10

1. Let S be a compact surface in $\mathbb{R}^3$, and let $p \in S$ be the point of S that is furthest from the origin. Prove that the Gaussian curvature at p is positive.

2. Let S be a compact surface in $\mathbb{R}^3$, and suppose that its Euler characteristic is $\chi(S) \leq 0$. Prove that there must be points on S where the Gaussian curvature is positive, zero, and negative.

3. Let $\gamma(t)$ be a simple closed curve in the $x - z$ plane, whose x–coordinate is always positive. Let S be the surface of revolution obtained by rotating γ about the z–axis.

(a) Compute the Euler characteristic of S.

(b) What standard surface is S homeomorphic to?

4. Let S be a surface homeomorphic to a sphere, such that the Gaussian curvature of S is everywhere positive. Let γ_1 and γ_2 be simple closed geodesics on S, such that the interior of γ_1 is region R_1 and the interior of γ_2 is region R_2. Prove that regions R_1 and R_2 must intersect. *Hint:* if they do not intersect, think about what's left of S after removing both R_1 and R_2.