Math 4061 Homework 7

Due Thursday, 4/22/10

1. Let S be the hyperboloid $z = 2xy$.

(a) Prove that at the origin, S has mean curvature $H = 0$ and Gaussian curvature $K = -4$.

(b) Let T be a surface obtained by turning S about the origin until the principal vectors at $(0,0,0)$ become tangent to the x-axis and y-axis. Describe a parametrization of T.

2. Let S the ellipsoid described by the equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

(a) Let $R \subset S$ be the region where $x \geq 0, y \geq 0, z \geq 0$. Sketch the image of R under the Gauss map. What is its area?

(b) Compute $\iint_R K\, dA = \iint_R K\, |\sigma_u \times \sigma_v|\, du\, dv$, without doing any calculus. *Hint:* examine the proof of Theorem 7.1 in the book.

(c) Prove that the integral of Gaussian curvature over the entire surface is $\iint_S K\, dA = 4\pi$.

3. Let $\gamma(t) = (x(t), z(t))$ be a unit-speed curve, such that $x(t) > 0$ for all t. Let S be the surface obtained by revolving γ about the z–axis.

(a) Prove that $\gamma(t) = (x(t), 0, z(t))$ is a geodesic in S.

(b) Let t_0 be a number such that $z(t)$ reaches a local maximum at t_0, and let $\delta(s)$ be the circle obtained by revolving the point $(x(t_0), z(t_0))$ about the z–axis. Is $\delta(s)$ a geodesic in S?

Hint: this should only need the definition of a geodesic. Drawing pictures will help!

4. The Little Prince lives on a planet whose radius is $5m$, and tends to a rose garden whose area is $100m^2$. If the rose garden is an equilateral triangle on this planet, what is the angle at each corner?