Math 4061 Homework 6

Due Thursday, 4/8/10

1. Consider the paraboloid S, defined by the equation $z = x^2 + y^2$.

(a) Compute the principal curvatures at a point (x, y, z) of S.

(b) What are the umbilic points, where $\kappa_1 = \kappa_2$?

(c) Let $\gamma(t) = (\cos t, \sin t, 1)$ be a circle on S. What is the geodesic curvature of γ?

2. Let S be a surface, whose principal curvatures satisfy $|\kappa_1| \leq 1$, $|\kappa_2| \leq 1$ at every point of S. Does every curve γ on S have to have curvature $\kappa \leq 1$?

3. Let p be a point on a surface S, whose principal curvatures are $\kappa_1 > 0$ and $\kappa_2 = -\kappa_1$. An *asymptotic direction* at p is a unit vector T tangent to S, such that the normal curvature in the direction of T is zero. Prove that (up to sign) there are exactly two asymptotic directions at p, and that these directions are perpendicular.

4. Let S be a surface P a plane. Suppose that S and P are tangent to each other along a curve γ. Prove that every point of γ is either a parabolic point or a planar point for S.