

Math 4061 Homework 3

Due Thursday, 2/18/10

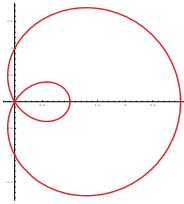
1. For each of the following subsets of \mathbb{R}^3 , decide (i) whether that set is a surface, and (ii) whether it is a smooth surface. Justify your answer (i.e., if the set is a surface, write down an atlas of coordinate patches).

(a) D is the unit disk in the x - y plane: $D = \{(x, y, 0) : x^2 + y^2 < 1\}$.

(b) C is a cone: $C = \{(x, y, z) : x^2 + y^2 = z^2, z \geq 0\}$.

(c) S is a spiral staircase: $S = \{(r \cos \theta, r \sin \theta, \theta) : r > 0, \theta \in \mathbb{R}\}$.

(d) F is a vertical “cylinder” whose projection to the x - y plane is the curve drawn below:
 $F = \{(r \cos \theta, r \sin \theta, z) : r = 1 + 2 \cos \theta\}$.



2. Let $f(x, y, z) = xyz^2$.

(a) Locate the critical points and critical values of f .

(b) For what values of c is the level set $f(x, y, z) = c$ a regular surface?

3. Let $G = \{(x, y, z) : x^2 + y^2 = 1\}$ be a cylinder, and let $H = \{(x, y, z) : x^2 + y^2 = 1 + z^2\}$ be a hyperboloid (see page 66 of the book for a picture). Find a diffeomorphism from G to H .
Hint: think about these surfaces in cylindrical coordinates.

4. Consider Definition 4.1 on page 60, where a surface is defined. Write down the analogous definition for a curve. Is your definition equivalent to the definition of a parametrized curve?