Math 4061 Homework 2

Due Thursday, 2/4/10

1. Let $\gamma : \mathbb{R} \to \mathbb{R}^2$ be a smooth unit-speed curve in the plane. Let δ be the same curve traversed backwards: that is, $\delta(-t) = \gamma(t)$. How does the signed curvature κ_s at $\gamma(t)$ relate to the signed curvature at $\delta(-t)$?

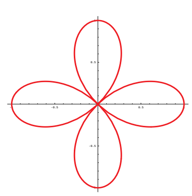
2. Let $\gamma : \mathbb{R} \to \mathbb{R}^2$ be a curve whose unsigned curvature at time t is $\kappa(t) = \dfrac{1}{t^2+1}$.

(a) Find a unit–speed parametrization of γ, and sketch the curve. *Hint:* a table of integrals, such as the one in a typical calculus book, can be helpful here.

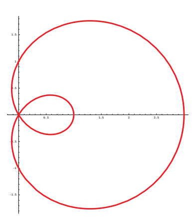
(b) Is the solution to (a) unique up to rigid motions? Why or why not? *Hint:* think about the distinction between signed and unsigned curvature.

3. For each of the following closed curves, compute the winding number $\frac{1}{2\pi}\int_0^a \kappa_s \, ds$. Recall: you can do this by inspection, without doing derivatives & integrals!

(a) $r = \cos 2\theta$. Parametrization: $\gamma(t) = (\cos 2t \cos t, \cos 2t \sin t)$.



(b) $r = 1 + 2\cos\theta$. Parametrization: $\gamma(t) = ((1+2\cos t)\cos t, (1+2\cos t)\sin t)$.



4. Let $\gamma : (a,b) \to \mathbb{R}^2$ be a curve with the property that all normal vectors at all points of γ are directed toward a single point of $\mathbb{R}^2$. In other words, for every $s \in (a,b)$, there is a scalar $\lambda(s)$ such that

$$\gamma(s) + \lambda(s)N(s) = v,$$

where v is a constant vector that does not depend on s. Prove that γ is a contained in a circle.

5. Consider the curve $\gamma(t) = (t^3, \sqrt{3}\,t^2, 2t)$. Compute the scalar quantities κ, τ and the vectors T, N, B, and check that the Frenet–Serret equations are satisfied.