

Math 4061 Homework 1

Due Thursday, 1/28/10

1. For $t \in \mathbb{R}$, let $\delta(t)$ be a parametrized curve which does not pass through the origin. Let t_0 be a time at which δ comes closest to the origin. Prove that $\delta(t_0) \cdot \delta'(t_0) = 0$.

2. Let γ be a parametrized curve such that $\gamma''(t) = 0$ for all t . What can you say about the shape of γ ?

3. Consider the graph of the absolute value function $y = |x - 1|$ in \mathbb{R}^2 .

- Find a parametrization of this curve.
- Find a four-times-differentiable parametrization of this curve.
- Does there exist a regular parametrization?

4. Consider a 3-dimensional logarithmic spiral, parametrized by

$$\alpha(t) = (e^{-t} \cos t, e^{-t} \sin t, e^{-t}).$$

- Compute the arclength between $\alpha(0)$ and $\alpha(r)$.
- Prove that the total length of α , as t ranges in $[0, \infty)$, is finite.

5. Consider the parametric curve $\gamma : [1, \infty) \rightarrow \mathbb{R}^2$, parametrized by

$$\gamma(t) = \left(\frac{\sin t}{t^2}, \frac{1}{t} \right).$$

- Does γ have a unit-speed reparametrization?
- Prove that the total length of γ is finite. *Hint:* Can you estimate the integral instead of computing it?