Math 1041, Quiz ~~2~~ 4

Thursday, 9/29/11

Name: Solutions

1. Let $f(x) = 4x\sqrt{x} + \sqrt{\frac{3}{x}}$. Compute $f'(x)$.

$$f(x) = 4x \cdot x^{1/2} + \frac{\sqrt{3}}{\sqrt{x}}$$

$$= 4x^{3/2} + \sqrt{3} \cdot x^{-1/2}$$

$$f'(x) = 4 \cdot \frac{3}{2} x^{1/2} + \sqrt{3} \cdot \frac{-1}{2} \cdot x^{-3/2}$$

$$= 6x^{1/2} - \frac{\sqrt{3}}{2} x^{-3/2}$$

Note: this is much easier if you simplify $f(x)$ first!

2. Let $g(x) = ke^x + 6/x$. If the tangent line to $g(x)$ at the point $(1, g(1))$ has slope exactly 5, what is the value of k?

Interpretation: the problem says that when $x = 1$, $g'(x) = 5$. In symbols, $g'(1) = 5$.

Now, differentiate $g(x)$.

$$g'(x) = ke^x - 6x^{-2}.$$

$$g'(1) = ke^1 - 6(1)^{-2} = ke - 6.$$

Since $g'(1) = 5$, we have

$$ke - 6 = 5$$

$$ke = 11$$

$$\boxed{k = 11/e}$$

3. Let $y = \dfrac{7 - x\,e^x}{x + e^x}$. Compute the derivative $\dfrac{dy}{dx}$.

$$y' = \frac{(x+e^x)\cdot\frac{d}{dx}(7 - xe^x) - (7 - xe^x)\frac{d}{dx}(x+e^x)}{(x+e^x)^2}$$

$$= \frac{(x+e^x)(-e^x - xe^x) - (7 - xe^x)(1+e^x)}{(x+e^x)^2}.$$

4. Suppose that $f(x)$ is a differentiable function, such that $f(4) = 4$ and $f'(4) = -3$. If $g(x) = \dfrac{8 + x\,f(x)}{\sqrt{x}}$, compute $g'(4)$.

$$g(x) = \frac{8}{\sqrt{x}} + \frac{x}{\sqrt{x}} f(x)$$

$$= 8x^{-1/2} + x^{1/2} f(x)$$

Again, simplifying original function is very helpful!

$$g'(x) = 8\cdot\frac{-1}{2}x^{-3/2} + \frac{1}{2}x^{-1/2}f(x) + x^{1/2}f'(x)$$

$$g'(4) = -4(4)^{-3/2} + \frac{1}{2}(4)^{-1/2}\cdot 4 + (4)^{1/2}\cdot(-3)$$

$$= -4\left(\frac{1}{8}\right) + \frac{1}{2}\cdot\frac{1}{2}\cdot 4 + 2(-3)$$

$$= -\frac{1}{2} + 1 - 6$$

$$= -5.5 = \frac{-11}{2}.$$

OR:

By the quotient rule: $g'(x) = \dfrac{\sqrt{x}\,(xf'(x) + f(x)) - (8 + xf(x))\cdot\frac{1}{2\sqrt{x}}}{\sqrt{x}\cdot\sqrt{x}}$

$= \ldots = -\dfrac{11}{2}.$