Math 1041, Quiz 2

Thursday, 9/15/11

Name: Solutions

1. Compute $\lim_{x\to-\infty} \frac{e^x}{e^x+3}$. $= \dfrac{\lim_{x\to-\infty} e^x}{\lim_{x\to-\infty} e^x+3} = \dfrac{0}{0+3} = 0.$

Another way: $\lim_{x\to-\infty} \dfrac{e^x}{e^x+3} \cdot \dfrac{1/e^x}{1/e^x} = \lim_{x\to-\infty} \dfrac{e^x/e^x}{e^x/e^x+3/e^x}$

$= \lim_{x\to-\infty} \dfrac{1}{1+3/e^x} = \dfrac{1}{\infty} = 0$, since $\lim_{x\to-\infty} \dfrac{3}{e^x} = \infty$.

Note: $\lim_{x\to\infty} \dfrac{e^x}{e^x+3} = \lim_{x\to\infty} \dfrac{e^x}{e^x+3} \cdot \dfrac{1/e^x}{1/e^x} = \lim_{x\to\infty} \dfrac{1}{1+3/e^x} = \dfrac{1}{1+0} = 1.$

But this is for $x \to +\infty$.

2. What are the horizontal and vertical asymptotes of $f(x) = \frac{x}{x^2-3}$?

Horizontal: $\lim_{x\to\infty} \dfrac{x}{x^2-3} = \lim_{x\to\infty} \dfrac{x/x^2}{x^2/x^2 - 3/x^2}$

$= \lim_{x\to\infty} \dfrac{1/x}{1-3/x^2}$

$= \dfrac{\lim_{x\to\infty} 1/x}{\lim_{x\to\infty} 1-3/x^2}$

$= 0/1 = 0.$

$\lim_{x\to-\infty} \dfrac{x}{x^2-3} = 0$, also (same calculation).

So we have a horizontal asymptote at $y=0$.

There are vertical asymptotes at $x = \pm\sqrt{3}$, since $(x^2-3) \to 0$ as $x \to \pm\sqrt{3}$.

3. If $g(x) = \sqrt{x}$, compute $g'(x)$ using the definition of the derivative.

$$g'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}.$$

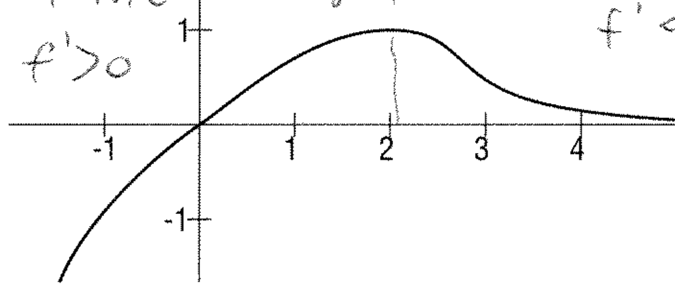
4. The graph of $f(x)$ is shown below.

f increasing

f'>0

f decreasing

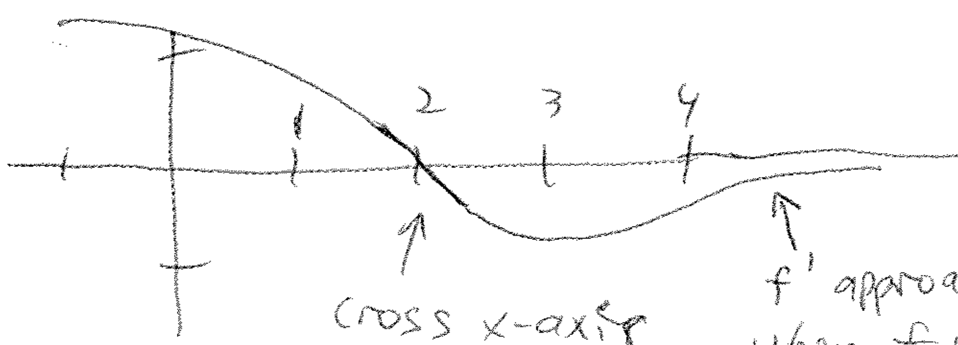
f'<0.



a) On what interval(s) is $f'(x)$ positive? On what interval(s) is $f'(x)$ negative?

$f'(x) > 0$ on $(-\infty, 2)$, when f is increasing

b) Sketch a graph of $f'(x)$. $f'(x) < 0$ on $(2, \infty)$, when f is decreasing.



cross x-axis at $x=2$

f' approaches 0 when f is nearly flat.