Math 1041, Quiz 2

Thursday, 9/15/11

Name: Solutions

1. Evaluate $\lim_{h\to 0} \frac{(2+h)^3-8}{h}$. $= \lim_{h\to 0} \frac{(h^3+6h^2+12h+8)-8}{h}$

$$= \lim_{h\to 0} \frac{h^3+6h^2+12h}{h}$$

$$= \lim_{h\to 0} h^2+6h+12$$

$$= 12.$$

Note: You need to write the "$=\lim_{h\to 0} \ldots$" at each step to get full credit on exams.

2. Suppose that $\cos(x) \le f(x) \le x^2+1$ for all x. What can you conclude about $\lim_{x\to 0} f(x)$? Explain.

$\lim_{x\to 0} \cos(x) = 1$, because cos is continuous.

$\lim_{x\to 0} (x^2+1) = 0^2+1 = 1$, because (x^2+1) is cont.

Thus, by the squeeze theorem,

$1 \le \lim_{x\to 0} f(x) \le 1$, i.e. $\lim_{x\to 0} f(x) = 1$.

(For full credit on exams: mention squeeze theorem, and write English explanation.)

3. Let $g(x) = \begin{cases} \ln(x), & x < 1, \\ \sin(\pi x), & x \geq 1. \end{cases}$ Where is g continuous?

Answer: g is continuous for all $x > 0$.
Note that when $x \leq 0$, $\ln(x)$ is not defined.
when $0 < x < 1$, $g(x) = \ln(x)$ is continuous.
when $x > 1$, $g(x) = \sin(\pi x)$ is continuous.
At $x = 1$, $\lim_{x \to 1^-} \ln(x) = \ln(1) = 0$.
$\lim_{x \to 1^+} \sin(\pi x) = \sin(\pi) = 0$.
Thus $g(1) = \lim_{x \to 1} g(x) = 0$, so g is continuous at $x = 1$.

4. In 1995, I had \$100 in my bank account, while LeBron James had \$0. In 2011, I have under \$10,000 in my bank account, while LeBron has over \$10 million. Can you conclude that at some time between 1995 and 2011, LeBron and I had the same amount of money in the bank? Why or why not?

Answer: NO, because bank account balances are not continuous. There is a jump discontinuity each time LeBron deposits a check.
So he can deposit a check for e.g. \$100,000 and jump right over me.

If the bank account balance was a continuous function, then the intermediate value theorem would say that LeBron has to pass me at some moment in time, and at that moment we would have equal balances.

Moral: it's important to check that a theorem applies.