2.2 # 32

Determine the infinite limit

$$\lim_{x \to 5^-} \frac{x+1}{x-5} = \frac{5+1}{0^-} = \frac{6}{0^-} = -\infty$$

2.3 # 1dc

Given that $\lim_{x \to 2} f(x) = 4$ and $\lim_{x \to 2} g(x) = -2$

find the limits that exist. If the limit d.n.e., explain why.

d) $$\lim_{x \to 2} \frac{3f(x)}{g(x)} = \frac{\lim_{x \to 2} 3f(x)}{\lim_{x \to 2} g(x)} = \frac{3 \lim_{x \to 2} f(x)}{-2} = \frac{3(4)}{-2} = -6$$

(using limit laws)

c) $$\lim_{x \to 2} \sqrt{f(x)} = \sqrt{\lim_{x \to 2} f(x)} = \sqrt{4} = 2$$

(since $\sqrt{x}$ is continuous at 2)

2.3 #23

Evaluate the limit, if it exists

$$\lim_{h\to 0} \frac{\left(\frac{1}{(x+h)^2} - \frac{1}{x^2}\right)\cdot (x+h)^2(x^2)}{h \cdot (x+h)^2(x^2)}$$

$$= \lim_{h\to 0} \frac{x^2-(x+h)^2}{h(x+h)^2x^2} = \lim_{h\to 0} \frac{\cancel{x^2}-\cancel{x^2}-2xh-h^2}{h(x+h)^2x^2}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}(x+h)^2x^2} = \frac{-2x-0}{(x+0)^2x^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

2.5 # 49b

Which of the following functions has a removable discontinuity at a? If the discontinuity is removable, find a function g that agrees with f for $x \neq a$ and is continuous at a

b) $$f(x) = \frac{x^3 - x^2 - 2x}{x-2} \quad , \quad a=2$$

If the zero a=2 in the denominator cancels, the function f has a removable discontinuity at a=2.

$$f(x) = \frac{x(x^2-x-2)}{(x-2)} = \frac{x(x+1)(x-2)}{(x-2)}$$

removable!

$\therefore$ f has a removable discontinuity at a = 2

$$g(x) = x(x+1)$$

g agrees with f everywhere except at a = 2, g is continuous

Ch 2 Review Ex. #13

Find the limit

$$\lim_{x\to\infty} \frac{\sqrt{x^2-9}}{2x-6} \cdot \frac{1/\sqrt{x^2}}{1/x} = \lim_{x\to\infty} \frac{\sqrt{1-\cancel{9/x^2}}}{2-\cancel{6/x}} = \frac{\sqrt{1}}{2} = \frac{1}{2}$$

3.5 #52

Find the derivative of the function.
Simplify where possible

$$g(x) = \arccos \sqrt{x}$$

$$g'(x) = -\frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}}$$

$$= -\frac{1}{2\sqrt{(1-x)x}}$$

3.6 #34 (if time)

Find an equation of the tangent line to the curve at the given point

$$y = x^2 \ln x, \qquad (1,0)$$

need:

$m = y'\Big|_{x=1} = 1$

$(x_1, y_1) = (1,0)$

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x-1)$

$y = x - 1$

$$y' = \cancel{x^2}^{\,x} \cdot \frac{1}{\cancel{x}} + \ln x \cdot 2x$$

$$m = y'\Big|_{x=1} = 1 + \underbrace{\cancel{\ln 1}}_{0} \cdot 2 \cdot 1 = 1$$

Ch 3 Review Exercises #31

Calculate y'

$$y = x \tan^{-1}(4x)$$

$$y' = x \cdot \frac{1}{1+(4x)^2} \cdot 4 + \tan^{-1}(4x)$$

Ch 3 Review Ex. #108

Express the limit as a derivative and evaluate

$$\lim_{\theta \to \pi/3} \frac{\cos\theta - 0.5}{\theta - \pi/3} = f'(a) = \lim_{\theta \to a} \frac{f(\theta) - f(a)}{\theta - a}$$

$f(\theta) = \cos\theta \qquad f(a) = \cos\frac{\pi}{3} = \frac{1}{2}$

$a = \frac{\pi}{3}$

$f'(\theta) = -\sin\theta$

$$f'(a) = f'\left(\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2} = \lim_{\theta \to \frac{\pi}{3}} \frac{\cos\theta - 0.5}{\theta - \pi/3}$$

4.3 #8

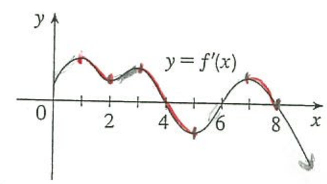
The graph of the first derivative f' of a function f is shown.

a) On what intervals is f increasing? Explain

b) At what values of x does f have a local maximum or minimum? Explain

c) On what intervals is f concave upward or concave downward? Explain.

d) What are the x-coordinates of the inflection points of f? Why?



a) f incr $\Rightarrow$ $f'(x) > 0$ $\Rightarrow$ on $(0,4)$ and $(6,8)$

f dcr $\Rightarrow$ $f'(x) < 0$ $\Rightarrow$ on $(4,6)$, $(8,\infty)$

b) By FDT,
f' changes from $\ominus$ to $\oplus$ at $x=6$; Thus f has a local min at $x=6$

f' changes from $\oplus$ to $\ominus$ at $x=4$ + $x=8$. Thus f has a local max at $x=4$ + $x=8$

c) f is concave up $\Rightarrow$ $f''(x)>0$ $\Rightarrow$ $f'(x)$ incr $\Rightarrow$ $(0,1)$, $(2,3)$, $(5,7)$

f is concave down $\Rightarrow$ $f''(x)<0$ $\Rightarrow$ $f'(x)$ dcr $\Rightarrow$ $(1,2)$, $(3,5)$, $(7,\infty)$

d) The x-coord. of I.P. are $x = 1, 2, 3, 5, 7$

b/c f changes concavity there.

4.4 #31

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

$$\lim_{x \to 0} \frac{\sin^{-1} x}{x} = \text{"}\frac{0}{0}\text{"} \text{, indet. form}$$

$$\overset{L'H}{=} \lim_{x \to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1} = \lim_{x \to 0} \frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-0}} = 1$$

4.4 # 75

What happens if you try to use l'Hospital's Rule to find the limit? Evaluate the limit using another method.

Try L'H:

$$\lim_{x\to\infty} \frac{\overset{\nearrow\infty}{x}}{\underset{\searrow\infty}{\sqrt{x^2+1}}} = \frac{\infty}{\infty}\text{, indet' form}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{1}{\frac{1}{2\sqrt{x^2+1}}\cdot 2x} = \lim_{x\to\infty} \frac{\overset{\nearrow\infty}{\sqrt{x^2+1}}}{\underset{\searrow\infty}{x}} = \frac{\infty}{\infty}\text{, indet form}$$

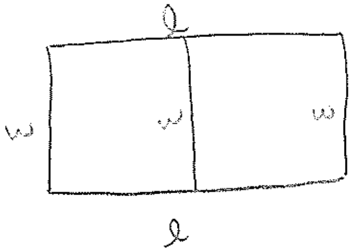
$$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{2\sqrt{x^2+1}}\cdot 2x}{1} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}\text{, back to the original!}$$

More elementary method:

$$\lim_{x\to\infty} \frac{x \cdot 1/x}{\sqrt{x^2+1}\cdot 1/\sqrt{x^2}} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\underset{0}{\cancel{1/x^2}}}} = \frac{1}{\sqrt{1+0}} = 1$$

4.7 #13

A farmer wants to fence in an area of 1.5 million ft^2 in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?



variables: l, w (in ft)

relationship between the variables: $l\,w = 1.5 \cdot 10^6 \ ft^2$

Quantity to be optimized (length of fencing): $Q = 2l + 3w$

Q in terms of on variable: $l = \dfrac{1.5 \cdot 10^6}{w}$

$$Q(w) = 2 \cdot 1.5 \cdot 10^6 \, w^{-1} + 3w$$

where is Q min? $Q'(w) = -3\,10^6\,w^{-2} + 3$

Critical #s:

- $Q'(w) = 0 \Rightarrow -3 \cdot 10^6 w^{-2} + 3 = 0 \Rightarrow 3 = \dfrac{3\,10^6}{w^2} \Rightarrow w^2 = 10^6$

  $w = 10^3$

- $Q'(w)$ dne $\Rightarrow w = 0$ (not in domain)

Is Q min at $w = 10^3$?

$Q''(w) = -3 \cdot 10^6 \cdot (-2w^{-3}) = 6\,10^6 \cdot w^{-3}$

$Q''(10^3) = 6 \cdot 10^6 (10^3)^{-3} = 6 \cdot 10^{-3}$, positive.

By second derivative test, (Q is concave up at $w = 10^3$), Q has a local min at $w = 10^3$.

when $w = 10^3$,

$l = \dfrac{1.5 \cdot 10^6}{10^3} = 1.5 \cdot 10^3 = 1{,}500$

The cost of the fencing is minimum when $w = 1000$ ft and $l = 1500$ ft (interior fencing has length $w = 1000$ ft)

4.9 # 37

Find f:

$f'(t) = \sec t\,(\sec t + \tan t)$

$-\pi/2 < t < \pi/2$

$f(\pi/4) = -1$

$f'(t) = \sec^2 t + \sec t \tan t$

$f(t) = \tan t + \sec t + C$

$f(\frac{\pi}{4}) = -1 = \tan(\pi/4) + \sec(\pi/4) + C$

$\Rightarrow \quad -1 = 1 + \sqrt{2} + C$

$-2 - \sqrt{2} = C$

$f(t) = \tan t + \sec t - 2 - \sqrt{2}$

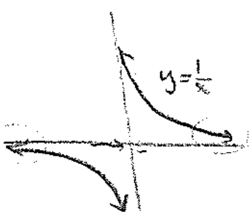
Ch 4 Review Ex #40b (only limits)

Find limits of $f(x) = \dfrac{1}{1+e^{1/x}}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1}{1+e^{1/x}} = \lim_{y\to 0^+} \frac{1}{1+e^{y}} = \frac{1}{1+e^{0}} = \frac{1}{1+1} = \frac{1}{2}$$

Sub: $y = \frac{1}{x}$

as $x \to \infty$, $y \to 0^+$



$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{1}{1+e^{1/x}} = \lim_{y\to 0^-} \frac{1}{1+e^{y}} = \frac{1}{1+e^{0}} = \frac{1}{1+1} = \frac{1}{2}$$

Sub: $y = 1/x$

as $x \to -\infty$, $y \to 0^-$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{1}{1+e^{1/x}} = \lim_{y\to \infty} \frac{1}{1+\underbrace{e^{y}}_{\infty}} = \frac{1}{\infty} = 0$$

Sub: $y = \frac{1}{x}$

as $x \to 0^+$, $y \to \infty$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{1}{1+e^{1/x}} = \lim_{y\to -\infty} \frac{1}{1+\underbrace{e^{y}}_{0}} = \frac{1}{1+0} = 1$$

Sub $y = 1/x$

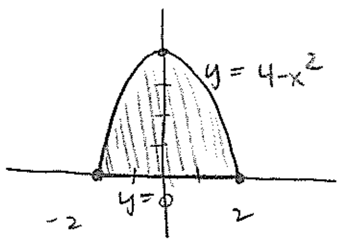
as $x \to 0^-$, $y \to -\infty$

5.3 # 47

Sketch the region enclosed by the given curves and calculate its area

$y = 4 - x^2$

$y = 0$



area of enclosed region

$$= \int_{-2}^{2} (4 - x^2)\, dx$$

$$= \left(4x - \frac{x^3}{3}\right)\Bigg|_{-2}^{2}$$

$$= \left(4 \cdot 2 - \frac{2^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right)$$

$$= 8 - \frac{8}{3} + 8 - \frac{8}{3} = \frac{3 \cdot 16}{3} - \frac{16}{3} = \frac{32}{3}$$

5.4 # 14

Find the general indefinite integral

$$\int \left(\frac{1+r}{r} \right)^2 dr$$

$$= \int (r^{-1} + 1)^2 \, dr = \int \left(r^{-2} + \frac{2}{r} + 1 \right) dr$$

$$= \frac{r^{-1}}{-1} + 2 \ln |r| + r + C$$

$$= \frac{-1}{r} + 2 \ln |r| + r + C$$

5.4 # 37

Evaluate the integral

$$\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\, d\theta$$

$$= \int_0^{\pi/4} \left(\frac{1}{\cos^2\theta} + 1\right) d\theta = \int_0^{\pi/4} (\sec^2\theta + 1)\, d\theta$$

$$= (\tan\theta + \theta)\Big|_0^{\pi/4} = \left(\tan\frac{\pi}{4} + \frac{\pi}{4}\right) - (\tan 0 + 0)$$

$$= 1 + \frac{\pi}{4}$$

5.6 #69

Evaluate the definite integral

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} = \int_1^4 \frac{1}{\sqrt{u}}\,du = \int_1^4 u^{-1/2}\,du$$

Sub:

$u = \ln x$

$du = \frac{1}{x}\,dx$

$x = e \Rightarrow u = \ln e = 1$

$x = e^4 \Rightarrow u = \ln(e^4) = 4 \cdot 1 = 4$

$$= \left.\frac{u^{1/2}}{1/2}\right|_1^4$$

$$= 2\left(4^{1/2} - 1^{1/2}\right)$$

$$= 2(2-1) = 2 \cdot 1 = 2$$

Ch 5 Review Ex #37

Evaluate the integral, if it exists

$$\int \frac{\sec\theta \tan\theta}{1+\sec\theta}\, d\theta = \int \frac{1}{u}\, du = \ln|u| + C$$

Sub:

$u = 1 + \sec\theta$

$du = \sec\theta \tan\theta \, d\theta$

$= \ln|1+\sec\theta| + C$