A6: Applications of Integration

1. Find the volume of the described solid S.

(**a**) The area $A(x)$ of the cross-section of S perpendicular to the x-axis and passing through the point x, for each x between 0 and 1, is $A(x) = \sin(\pi x)$.

(**b**) The base of S is the region between the curve $y = \dfrac{1}{x}$ and the x-axis bounded on the sides by $x = 1$ and $x = 2$. The cross-sections perpendicular to the x-axis are figures with the area $A = \frac{b^3}{12}$ where b is the length of the intersection of the cross-section with the base.

(**c**) The base of S is the region bounded by the lines $y = x$, $y = 3x$, $x = 1$, and $x = 2$. The cross-sections perpendicular to the x-axis are semidisks with the diameter in the base.

(**d**) The base of S is the region bounded by the curves $y = \sqrt{x}$, $y = \sqrt[3]{x}$, and the lines $y = 1$ and $y = 2$. The cross-sections perpendicular to the y-axis are squares.

A7: Techniques of Integration

1. The base of a solid is the region bounded by the curve $y = \sin x$ and the lines $y = x$ and $x = \pi/2$. Cross-sections perpendicular to the x-axis are isosceles right triangles with hypotenuse in the base. Sketch the base and find the volume of the described solid.

2. The region in the xy-plane is bounded by the curves $y = 2\cos x$, $y = \tan x$ and the lines $x = 0$, $x = \pi/4$.

(**a**) Sketch the region.

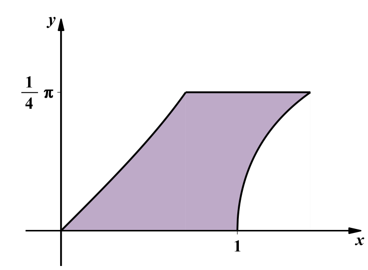
(**b**) Find the volume of the solid with this region as its base if its cross-sections perpendicular to the x-axis are squares.

(**c**) Find the volume of the solid obtained by rotating the region about the x-axis.

3. The region in the xy-plane is bounded by the curves $y = \arcsin x$, $y = \operatorname{arcsec} x$ and the lines $y = 0$, $y = \pi/4$.

(**a**) Find the volume of the solid with this region as its base if its cross-sections perpendicular to the y-axis are squares.

(**b**) Find the volume of the solid obtained by rotating the region about the y-axis.



A11: Infinite Sequences and Series

1. Determine whether the sequence $\{a_n\}$ converges or diverges. If it converges, find the limit.

(**a**) $a_n = \sqrt[n]{2n+1}$ (*Hint: you may use the fact that* $\lim_{n\to\infty} \sqrt[n]{n} = 1$.)

(**b**) $a_n = \sqrt[n]{3n^5 + n^2 + 1}$

(**c**) $a_n = \sqrt{3n^5 + n^2 + 1}$

2. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

(**a**) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, 3e^{n+2}}{4^n}$ (**b**) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 4^n}{5e^{n+2}}$ (**c**) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 4^n}{5e^{2n+2}}$

3. Find the $\boldsymbol{n}$th partial sum of the telescoping series. Then determine whether the series is convergent or divergent. If it is convergent, find its sum.

(**a**) $\displaystyle\sum_{n=1}^{\infty} (\cos n - \cos(n-1))$ (**b**) $\displaystyle\sum_{n=1}^{\infty} \left[\left(1 + \tfrac{1}{n}\right)^n - \left(1 + \tfrac{1}{n+1}\right)^{n+1}\right]$

4. The ***n*th** partial sum s_n of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = \ln(n+1) - \ln(2n+1)$. Find the term a_n (present it as a single logarithm) and determine whether the series is convergent or divergent. If it is convergent, find its sum.

5. Determine whether the series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3+\sin n}{\sqrt{n}}$ **(b)** $\displaystyle\sum_{n=1}^{\infty} \frac{3+\sin n}{n\sqrt{n}}$ **(c)** $\displaystyle\sum_{n=1}^{\infty} \frac{\left(1+\frac{3}{n}\right)^n}{e^n+n}$ **(d)** $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{3^n+5^n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{n^2}$ **(f)** $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{n^{2n}}$