Section 7.3

2. $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\,dx = 8\int \tan^3\theta\,\sec\theta\,d\theta = \frac{1}{3}(x^2+4)^{3/2} - 4\sqrt{x^2+4} + C$

4. $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx = 9\int \sin^2\theta\,d\theta = \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{1}{2}x\sqrt{9-x^2} + C$

8. $\displaystyle\int \frac{1}{t^2\sqrt{t^2-16}}\,dt = \frac{1}{16}\int \cos\theta\,d\theta = \frac{\sqrt{t^2-16}}{16t} + C$

12. $\displaystyle\int_0^2 \frac{dt}{\sqrt{4+t^2}}\,dt = \int_0^{\pi/4} \sec\theta\,d\theta = \ln(\sqrt{2}+1)$

Section 7.4

2. (a) $\displaystyle\frac{x-6}{x^2+x-6} = \frac{A}{x+3} + \frac{B}{x-2}$ (b) $\displaystyle\frac{x^2}{x^2+x+6} = 1 - \frac{x+6}{x^2+x+6}$

28. $\displaystyle\int \frac{x^2+6x-2}{x^2+6x^2}\,dx = \int \left(\frac{1}{x} + \frac{-1/3}{x^2} + \frac{1/3}{x^2+6}\right) = \ln|x| + \frac{1}{3x} + \frac{1}{3\sqrt{6}}\tan^{-1}\left(\frac{x}{\sqrt{6}}\right) + C$

64. $\displaystyle A = \int_1^2 \frac{1}{x^3+x}\,dx = \int_1^2 \frac{1}{x(x^2+1)}\,dx = \int_1^2 \left(\frac{1}{x} - \frac{x}{x^2+1}\right)\,dx = \frac{3}{2}\ln 2 - \frac{1}{2}\ln 5$

66a. $\displaystyle V = \pi\int_0^1 \left(\frac{1}{x^2+3x+2}\right)^2 dx = \pi\int_0^1 \left(\frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{2}{x+2} + \frac{1}{(x+2)^2}\right)\,dx = \pi\left(\frac{2}{3} + \ln(\frac{9}{16})\right)$

Section 7.5

16. $\displaystyle\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx = \frac{\pi}{8} - \frac{1}{4}$

Section 7.8

1. (a) An improper integral as $y = \dfrac{x}{x-1}$ has an infinite discontinuity at 1 and $\displaystyle\int_1^2 \frac{x}{x-1}\,dx = \lim_{t\to 1^+}\int_t^2 \frac{x}{x-1}\,dx$

(b) An improper integral due to an infinite interval of integration and $\displaystyle\int_0^\infty \frac{1}{1+x^3}\,dx = \lim_{t\to\infty}\int_0^t \frac{1}{1+x^3}\,dx$

(c) An improper integral due to an infinite interval of integration and

$$\int_{-\infty}^\infty x^2e^{-x^2}\,dx = \int_{-\infty}^0 x^2e^{-x^2}\,dx + \int_0^\infty x^2e^{-x^2}\,dx = \lim_{t\to-\infty}\int_t^0 x^2e^{-x^2}\,dx + \lim_{r\to\infty}\int_0^r x^2e^{-x^2}\,dx$$

(d) An improper integral as $y = \cot x$ has infinite discontinuity at 0 and $\displaystyle\int_0^{\pi/4} \cot x\,dx = \lim_{x\to 0^+}\int_t^{\pi/4} \cot x\,dx$

2. (a) A proper integral

(b) An improper integral as $y = \tan x$ has an infinite discontinuity at $\dfrac{\pi}{2}$

and $\displaystyle\int_0^{\pi} \tan x\, dx = \int_0^{\pi/2} \tan x\, dx + \int_{\pi/2}^{\pi} \tan x\, dx = \lim_{t\to(\pi/2)^-} \int_0^{t} \tan x\, dx + \lim_{R\to(\pi/2)^+} \int_R^{\pi} \tan x\, dx$

(c) An improper integral as $y = \dfrac{1}{x^2 - x - 2} = \dfrac{1}{(x-2)(x+1)}$ has an infinite discontinuity at $x = -1$

and $\displaystyle\int_{-1}^{1} \frac{1}{x^2 - x - 2}\, dx = \lim_{t\to(-1)^+} \int_t^{1} \frac{1}{x^2 - x - 2}\, dx$

(d) An improper integral due to infinite interval of integration and $\displaystyle\int_0^{\infty} e^{-x^2}\, dx = \lim_{t\to\infty} \int_0^{t} e^{-x^2}\, dx$

24. Integral converges and $\displaystyle\int_e^{\infty} \frac{1}{x(\ln x)^2}\, dx = \lim_{t\to\infty} \int_e^{t} \frac{1}{x(\ln x)^2}\, dx = 1$.

45. An infinite volume as $V = \pi \displaystyle\int_0^{pi/2} \sec^4 x\, dx = \lim_{t\to\left(\frac{\pi}{2}\right)^-} \int_0^{t} \sec^4 x\, dx = \infty.$

50. Integral diverges by the Comp. Theo. as $\dfrac{1+\sin^2 x}{\sqrt{x}} \geq \dfrac{1}{\sqrt{x}}$ and $\displaystyle\int_1^{\infty} \frac{1}{\sqrt{x}}\, dx$ is divergent.

52. Integral Converges by the Comp. Theo. as $\dfrac{\arctan x}{2+e^x} < \dfrac{\pi/2}{e^x} < \dfrac{2}{e^x}$ and $\displaystyle\int_0^{\infty} \frac{2}{e^x}\, dx = 2$ converges.

Review Chapter 7

TRUE-FALSE. **2.** True **4.** False. Correct form is $\dfrac{A}{x} + \dfrac{Bx+C}{x^2+4}$ **6.** True **14.** False

Exercise: 10. $\dfrac{1}{12}\pi^{3/2}$

18. $\displaystyle\int \frac{x^2+8x-3}{x^(x+3)}\, dx = \int \left(\frac{3}{x} - \frac{1}{x^2} - \frac{2}{x+3}\right) = 3\ln|x| + \frac{1}{x} - 2\ln|x+3| + C$

Section 11.1

26. Converges to 2 **30.** Converges ty 0 **32.** Converges to -1 **38.** Converges to 1

48. Converges to 1 **50.** Converges to 0 **56.** Converges to 0

Section 11.2

4. Sum $= \displaystyle\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n^2-1}{4n^2+1} = \frac{1}{4}$. Series converges.

22. $a = \dfrac{5}{\pi}$ and $r = \dfrac{1}{\pi}$. Geometric series converges and Sum$= \dfrac{5}{\pi - 1}$.

24. $a = 3$ and $r = -\dfrac{3}{2}$. Geometric series diverges.

26. $a = 4$ and $r = \dfrac{4}{3}$. Geometric series diverges

34. Divergent as it is sum of a convergent and a divergent geometric series

46. Partial sum is $S_n = \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{n+4}}$ and $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n = \frac{1}{2}$, Series converges

A11 3a. $S_n = -1 + \cos n$ and $\lim_{n\to\infty} S_n = d.n.e$ and series diverges.

Section 11.3

8. Series converges by the Integral test as the integral $\int_1^{\infty} x^2\, e^{-x^3}\, dx = \frac{1}{3e}$ converges.

Section 11.4

2. (a) $\sum a_n$ diverges by the Comparison Test We can't say anything about $\sum_{n=1}^{\infty} a_n$.

8. Divergent By CT, LCT and Div. test **12:** Convergent LCT

24. Divergent by LCT and the Div. Test

A11 5. (a) Series diverges by the Comparison test with $b_n = \frac{1}{\sqrt{n}}$.

(b) Series converges by the Comparison test with $b_n = \frac{1}{n\sqrt{n}} = \frac{1}{n^{3/2}}$.

Section 11.5

6. Convergent by the Alt. Series Test.

Section 11.6

38. Series converges conditionally as $\sum_{n=1}^{\infty} |a_n|$ diverges by the integral test and $\sum_{n=1}^{\infty} a_n$ converges by the Alt. Series Test.

Review Chapter 11

2. Sequence converges to 0 **6.** Sequence converges to 0 **8.** Sequence converges to 0