

A6: Applications of Integration

1. Find the volume of the described solid S .

(a) The area $A(x)$ of the cross-section of S perpendicular to the x -axis and passing through the point x , for each x between 0 and 1, is $A(x) = \sin(\pi x)$.

(b) The base of S is the region between the curve $y = \frac{1}{x}$ and the x -axis bounded on the sides by $x = 1$ and $x = 2$. The cross-sections perpendicular to the x -axis are figures with the area $A = \frac{b^3}{12}$ where b is the length of the intersection of the cross-section with the base.

(c) The base of S is the region bounded by the lines $y = x$, $y = 3x$, $x = 1$, and $x = 2$. The cross-sections perpendicular to the x -axis are semidisks with the diameter in the base.

(d) The base of S is the region bounded by the curves $y = \sqrt{x}$, $y = \sqrt[3]{x}$, and the lines $y = 1$ and $y = 2$. The cross-sections perpendicular to the y -axis are squares.

A7: Techniques of Integration

1. The base of a solid is the region bounded by the curve $y = \sin x$ and the lines $y = x$ and $x = \pi/2$. Cross-sections perpendicular to the x -axis are isosceles right triangles with hypotenuse in the base.

Sketch the base and find the volume of the described solid.

2. The region in the xy -plane is bounded by the curves $y = 2 \cos x$, $y = \tan x$ and the lines $x = 0$, $x = \pi/4$.

(a) Sketch the region.

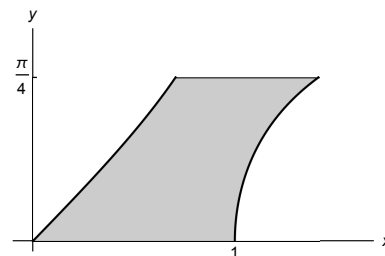
(b) Find the volume of the solid with this region as its base if its cross-sections perpendicular to the x -axis are squares.

(c) Find the volume of the solid obtained by rotating the region about the x -axis.

3. The region in the xy -plane is bounded by the curves $y = \arcsin x$, $y = \operatorname{arcsec} x$ and the lines $y = 0$, $y = \pi/4$.

(a) Find the volume of the solid with this region as its base if its cross-sections perpendicular to the y -axis are squares.

(b) Find the volume of the solid obtained by rotating the region about the y -axis.

**A11: Infinite Sequences and Series**

1. Determine whether the sequence $\{a_n\}$ converges or diverges. If it converges, find the limit.

(a) $a_n = \sqrt[3]{2n+1}$ (Hint: you may use the fact that $\lim_{n \rightarrow \infty} \sqrt[3]{n} = 1$.)

(b) $a_n = \sqrt[3]{3n^5 + n^2 + 1}$

(c) $a_n = \sqrt{3n^5 + n^2 + 1}$

2. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n 3e^{n+2}}{4^n}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^n}{5e^{n+2}}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^n}{5e^{2n+2}}$

3. Find the n th partial sum of the telescoping series. Then determine whether the series is convergent or divergent. If it is convergent, find its sum.

(a) $\sum_{n=1}^{\infty} (\cos n - \cos(n-1))$

(b) $\sum_{n=1}^{\infty} \left[\left(1 + \frac{1}{n}\right)^n - \left(1 + \frac{1}{n+1}\right)^{n+1} \right]$

4. The n th partial sum s_n of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = \ln(n+1) - \ln(2n+1)$. Find the term a_n

(present it as a single logarithm) and determine whether the series is convergent or divergent. If it is convergent, find its sum.

5. Determine whether the series converges or diverges.

(a) $\sum_{n=1}^{\infty} \frac{3 + \sin n}{\sqrt{n}}$

(b) $\sum_{n=1}^{\infty} \frac{3 + \sin n}{n\sqrt{n}}$

(c) $\sum_{n=1}^{\infty} \frac{\left(1 + \frac{3}{n}\right)^n}{e^n + n}$

(d) $\sum_{n=1}^{\infty} \frac{4^n}{3^n + 5^n}$

(e) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$

(f) $\sum_{n=1}^{\infty} \frac{(2n)!}{n^{2n}}$