Answer-Keys to Even Review Problems for Test 2

Section 7.3

2. $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\,dx = 8\int \tan^3\theta\,\sec\theta\,d\theta = \frac{1}{3}(x^2+4)^{3/2} - 4\sqrt{x^2+4} + C$

7. $\displaystyle\int_0^{16} \frac{1}{(x^2+256)^{3/2}}\,dx = \frac{1}{256}\int_0^{\pi/4} \cos\theta\,d\theta = \frac{1}{256\sqrt{2}}$

8. $\displaystyle\int \frac{1}{t^2\sqrt{t^2-16}}\,dt = \frac{1}{16}\int \cos\theta\,d\theta = \frac{\sqrt{t^2-16}}{16t} + C$

14. $\displaystyle\int_0^{1} \frac{1}{(x^2+1)^2}\,dx = \int_0^{\pi/4} \cos^2\theta\,d\theta = \frac{\pi}{8} + \frac{1}{4}$

Section 7.4

4a. $\displaystyle x^2 + \frac{A}{x-1} + \frac{B}{(x-1)^2}$

22. $\displaystyle \frac{x^3}{3} + \frac{1}{2}\ln(x^2+9) + \frac{2}{3}\tan^{-1}\frac{x}{3} + C$

Section 7.8

1. (a) Improper integral as $\dfrac{x}{x-1}$ has infinite discontinuity at $x=1$ and $\displaystyle\int_1^2 \frac{x}{x-1}\,dx = \lim_{R\to 1^+}\int_R^2 \frac{x}{x-1}\,dx$

(b) Infinite interval of integration so improper integral and $\displaystyle\int_0^\infty \frac{1}{1+x^3}\,dx = \lim_{R\to\infty}\int_0^R \frac{1}{1+x^3}\,dx$

(c) Infinite interval of integration so improper integral

and $\displaystyle\int_{-\infty}^{\infty} x^2e^{-x^2}\,dx = \lim_{R\to-\infty}\int_R^0 x^2e^{-x^2}\,dx + \lim_{b\to\infty}\int_0^b x^2e^{-x^2}\,dx$

(d) Improper integral as $\cot x$ has infinite discontinuity at $x=0$ and $\displaystyle\int_0^{\pi/4} \cot x\,dx = \lim_{R\to 0^+}\int_R^{\pi/4} \cot x\,dx$

2. (a) proper integral

(b) Improper integral as $\tan x$ has infinite discontinuity at $x = \dfrac{\pi}{2}$

and $\displaystyle\int_0^{\pi} \tan x\,dx = \lim_{b\to(\frac{\pi}{2})^-}\int_0^b \tan x\,dx + \lim_{R\to(\frac{\pi}{2})^+}\int_R^{\pi} \tan x\,dx$

(c) Improper integral as $\dfrac{1}{x^2-x-2} = \dfrac{1}{(x-2)(x+1)}$ has infinite discontinuity at $x=-1$

and $\displaystyle\int_{-1}^{1} \frac{1}{x^2-x-2}\,dx = \lim_{R\to(-1)^+}\int_R^1 \frac{1}{x^2-x-2}\,dx$

(d) Infinite interval of integration so improper integral and $\displaystyle\int_0^\infty e^{-x^3}\,dx = \lim_{b\to\infty}\int_0^b e^{-x^3}\,dx$

14. $\displaystyle = \int_1^\infty \frac{e^{-1/x}}{x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{e^{-1/x}}{x^2}\,dx = 1 - f1e$

50. Integral diverges by comparison with $g(x) = \dfrac{1}{\sqrt{x}}$

52. Integral converges by comparison with $g(x) = \dfrac{2}{e^x} = 2e^{-x}$

Section 11.1

26. Converges to 2 **28.** Converges to 3 **30.** Converges to 0

36. Diverges, limit dne **38.** Converges to 1 **40.** Converges to 0

50. Converges to 0 **56.** Converges to 0

Section 11.2

4. Series converges and Sum= $\lim_{n\to\infty} s_n = \frac{1}{4}$ **22.** Series converges and Sum= $\frac{5}{\pi - 1}$

24. Series diverges **26.** Divergent **34.** Series diverges

46. $S_n = \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{n+1}}$ and Sum= $\lim_{n\to\infty} S_n = \frac{1}{2}$. Series converges.

Section 11.3

4. Divergent P-series.

Section 11.4

2. (a) $\sum a_n$ diverges (b) $\sum a_n$ could be convergent or divergent

10. Series converges by the Comparison Test when $b_k = \frac{1}{k^2}$

A11: Sequences and Series

5. (a) Series diverges by comparison with $b_n = \frac{2}{\sqrt{n}}$ (b) Series converges by comparison wirh $b_n = \frac{4}{n^{3/2}}$

(d) Series converges by comparison with $b_n = \frac{4^n}{5^n} = \left(\frac{4}{5}\right)^n$

Section 11.5

6. Converges by the Alternative Series Test **10.** Converges by the Alternative Series Test

14. Divergent by the Test for Divergence

Section 11.6

4. $\sum a_n$ converges absolutely as $\sum |a_n|$ coverges by the comparison Test as $b_n = \frac{1}{n^3}$.

6. $\sum a_n$ converges conditionally as $\sum |a_n|$ diverges by the Comparison Test with $b_n = \frac{1}{n}$ and $\sum a_n$ converges by the AST

Chapter 11 Review

6. Converges to 0 **8.** Converges to 0 **14.** Converges by Alt. Series Test