MATH 1041 Supplementary Exercises

SE 2.3

1. Suppose $x \sin x \le f(x) \le \tan^2 x$ **for all** x **in** $(-\pi/2,\, \pi/2)$.

(a) **Find** $\lim_{x \to 0} x \sin x$ **and** $\lim_{x \to 0} \tan^2 x$.

(b) **State whether the Squeeze Theorem can or cannot be applied to find** $\lim_{x \to 0} f(x)$. **Justify your answer. Apply the theorem if it can be applied and find** $\lim_{x \to 0} f(x)$.

(c) **Find** $\lim_{x \to \pi/4} x \sin x$ **and** $\lim_{x \to \pi/4} \tan^2 x$.

(d) **State whether the Squeeze Theorem can or cannot be applied to find** $\lim_{x \to \pi/4} f(x)$. **Justify your answer. Apply the theorem if it can be applied and find** $\lim_{x \to \pi/4} f(x)$.

2. Suppose $\dfrac{3x^2 - 10x + 8}{x^2 - 2x} \le f(x) \le e^{x-2}$ **for all** x **in** $(0,\, 2) \cup (2,\, \infty)$.

(a) **Find** $\lim_{x \to 1} \dfrac{3x^2 - 10x + 8}{x^2 - 2x}$ **and** $\lim_{x \to 1} e^{x-2}$.

(b) **State whether the Squeeze Theorem can or cannot be applied to find** $\lim_{x \to 1} f(x)$. **Justify your answer. Apply the theorem if it can be applied and find** $\lim_{x \to 1} f(x)$.

(c) **Find** $\lim_{x \to 2} \dfrac{3x^2 - 10x + 8}{x^2 - 2x}$ **and** $\lim_{x \to 2} e^{x-2}$.

(d) **State whether the Squeeze Theorem can or cannot be applied to find** $\lim_{x \to 2} f(x)$. **Justify your answer. Apply the theorem if it can be applied and find** $\lim_{x \to 2} f(x)$.

SE 2.5

In Problems 1, 2, and 3, find the values of a **and** b **(or just** a**) that make the function** $f(x)$ **continuous everywhere in its domain.**

1. $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \textbf{if} \quad x < 2 \\ a & \textbf{if} \quad x = 2 \\ 5x - a + b & \textbf{if} \quad x > 2 \end{cases}$

2. $f(x) = \begin{cases} \frac{\sqrt{x}-\sqrt{3}}{x-3} & \textbf{if} \quad x \ne 3,\ x > 0 \\ a & \textbf{if} \quad x = 3 \end{cases}$

3. $f(x) = \begin{cases} e^{x-a} & \textbf{if} \quad x \ne 1 \\ 3 & \textbf{if} \quad x = 1 \end{cases}$

SE 2.6

1. Suppose $2\arctan x \le f(x) \le \dfrac{\pi x^2+4}{x^2+1}$ **for all** x**.**

(a) Find $\lim\limits_{x\to\infty} 2\arctan x$ **and** $\lim\limits_{x\to\infty} \dfrac{\pi x^2+4}{x^2+1}$**.**

(b) State whether the Squeeze Theorem can or cannot be applied to find $\lim\limits_{x\to\infty} f(x)$**. Justify your answer. Apply the theorem if it can be applied and find** $\lim\limits_{x\to\infty} f(x)$**.**

(c) Find $\lim\limits_{x\to 1} 2\arctan x$ **and** $\lim\limits_{x\to 1} \dfrac{\pi x^2+4}{x^2+1}$**.**

(d) State whether the Squeeze Theorem can or cannot be applied to find $\lim\limits_{x\to 1} f(x)$**. Justify your answer. Apply the theorem if it can be applied and find** $\lim\limits_{x\to 1} f(x)$**.**

SE 3.10

In Problems 1 and 2,

(a) find the linearization $L(x)$ **of the function** $f(x)$ **at the given value of** a **(please note that the linearization should be written in the form** $L(x) = f(a) + f'(a)(x-a)$ **and should NOT be rewritten in the form** $L(x) = Ax + B$**);**

(b) use $L(x)$ **to approximate** $f(x)$ **at the given value of** x**.**

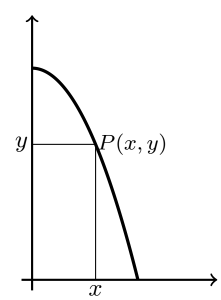
1. $f(x) = 4\arctan x$, $a = 1$. **Approximate** $f(1.01)$. **Round your answer to two decimal places.**

2. $f(x) = x\,e^{x-2}$, $a = 2$. **Approximate** $f(1.9)$.

3. Suppose $f(-1) = 4$ **and** $f'(-1) = -2$. **Find the linearization of** $f(x)$ **at** $a = -1$ **and use it to estimate** $f(-1.01)$**.**

SE 4.1

1. Let $f(x) = 4 - x^2$. **Consider a rectangle with vertices** $(0,0)$, $(x,0)$, (x,y), **and** $(0,y)$, **where the vertex** (x,y) **lies on the curve** $y = f(x)$, $0 \le x \le 2$ **(see the picture at right).**



Part I.

(a) Express the perimeter P **of the rectangle as a function of** x, $P(x)$**.**

(b) Find the absolute maximum and absolute minimum of $P(x)$ **on the interval** $0 \le x \le 2$**.**

(c) Find the dimensions of the rectangle with the largest perimeter.

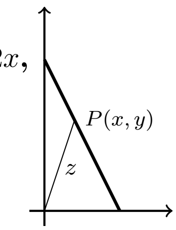
Part II.

(a) Express the area A of the rectangle as a function of x, $A(x)$.

(b) Find the absolute maximum and absolute minimum of $A(x)$ on the interval $0 \leq x \leq 2$.

(c) Find the dimensions of the rectangle with the largest area.

2. Let z denote the distance from the origin to the point (x, y) on the line $y = 2 - 2x$, $0 \leq x \leq 1$ (see the picture at right).



(a) Express z as a function of x, $z(x)$.

(b) Find the absolute maximum and absolute minimum of $z(x)$, $0 \leq x \leq 1$.

(c) Find the coordinates of the point on the segment $y = 2 - 2x$, $0 \leq x \leq 1$, that is closest to the origin.